The Earth's Outward Radiation Window and Equilibrium Climate Sensitivity

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Abstract

In assessing the impacts of climate change/global warming, the magnitude of the Earth's equilibrium climate sensitivity to carbon dioxide is an important parameter. The Earth's warming can come from the *direct* greenhouse effect of more anthropogenic atmospheric carbon dioxide, and it can also come from some *indirect effects* such as water vapor feedback. The fact that the outward infrared radiation can only pass through a restricted wave-length window set up by all the atmospheric greenhouse gases also contributes to the indirect effects. For the Earth this contribution partially ameliorates the direct greenhouse effect and other indirect effects.

Keywords: climate change, global warming, climate sensitivity, carbon dioxide.

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1 Introduction

The Earth receives radiant energy from the Sun, and maintains its long term energy balance by its own radiations outward into space. The Earth's long term *quasi-steady* energy balance equation is:

$$F^{out}_{earth}(T, X) = F^{in}_{solar} \approx F^{in}_{solar,o}, \tag{1}$$

where T is the Earth's average surface temperature in ^oK, X is the amount of carbon dioxide in the atmosphere, F^{in}_{solar} is the incoming solar flux (after properly averaging over latitudes/days/nights, and accounting for the Earth's albedo), and F^{out}_{earth} is the Earth's average outgoing flux. Both F^{in}_{solar} and F^{out}_{earth} are evaluated at the top of the Earth's atmosphere. Subscript "o" denotes the pre-industrial (average) value. The commonly accepted value for $F^{in}_{solar,o}$, often referred to as *solar irradiance*, is approximately 342 $watts/meter^2$. Even though the Earth definitely does not radiate as a grey body, nevertheless $F^{out}_{earth}(T, X)$ is often represented by (see Eq.(14) in §3 later):

$$F^{out}_{earth}(T, X) = \sigma\widehat{\epsilon}_{earth}(T, X)T^4, \tag{2}$$

where $\sigma = 5.67 \times 10^{-8}$ watts/$^oK^4 meter^2$ is the Stefan-Boltzmann constant, and $\widehat{\epsilon}_{earth}(T, X)$, the *effective emissivity*, represents all the atmospheric complications, including the *direct* effect of outward radiation absorptions (greenhouse effects) and all the relevant *indirect* effects. Setting the Earth's pre-industrial temperature T_o at 293 ^oK and equating $F^{out}_{earth}(T_o)$ to $F^{in}_{solar,o} =$ 342 $watts/meter^2$, the value of $\widehat{\epsilon}_{earth}(293\ ^oK, X_o)$ is found to be approximately 0.82.

When the direct effect on $\widehat{\epsilon}_{earth}(T, X)$ by post-industrial incremental atmospheric carbon dioxide absorptions is separately and explicitly accounted for, Eq.(1) can be rewritten as:

$$F^{out}_{earth}(T,X) = \sigma\widehat{\epsilon}_{earth}(T_o+\Delta T,X)(T_o+\Delta T)^4 \quad (3a)$$

$$= \sigma\widehat{\epsilon}_{indrct}(T_o+\Delta T,X_o)(T_o+\Delta T)^4 - \Delta F^{out}_{CO_2} \quad (3b)$$

$$\approx F^{out}_{earth}(T_o,X_o) = \sigma\widehat{\epsilon}_{earth}(T_o,X_o)T_o^4. \quad (3c)$$

where $\Delta F^{out}_{CO_2} > 0$ is the amount of outward radiation directly absorbed by the incremental atmospheric carbon dioxide, and ΔT is the total (direct and indirect) incremental temperature rise needed when $\Delta F^{out}_{CO_2} \neq 0$. When $X = X_o$, $\Delta F^{out}_{CO_2} = 0$. Thus $\widehat{\epsilon}_{earth}(T, X \to X_o) = \widehat{\epsilon}_{indrct}(T,X_o)$. All the *indirect* consequences of $\Delta F^{out}_{CO_2}$ on ΔT are the responsibility of the T dependence of $\widehat{\epsilon}_{indrct}(T,X_o)$ which is defined by Eq.(3b).

1.1 The Linearized ΔT Response to $\Delta F^{out}_{CO_2} \neq 0$

For $|\Delta F^{out}_{CO_2}| << F^{out}_{total,o}$, the first term on the right hand side of Eq.(3b) can be linearized to yield:

$$\overbrace{\frac{\Delta T}{T_o}}^{\text{fractional T change}} \approx \frac{1}{4+\mathcal{E}_{indrct}(T_o)} \overbrace{\frac{\Delta F^{out}_{CO_2}}{F^{out}_{total,o}}}^{\text{fractional } F^{out}_{earth} \text{ change}}, \quad (4)$$

where $\mathcal{E}_{indrct}(T_o)$ is shorthand for:

$$\mathcal{E}_{indrct}(T_o) \equiv \left(\frac{\partial \ln \widehat{\epsilon}_{indrct}}{\partial \ln T}\right)_o = \left(\frac{\partial \ln \widehat{\epsilon}_{earth}}{\partial \ln T}\right)_o. \quad (5)$$

Eq.(4) says the ΔT response for a given $\Delta F^{out}_{CO_2}$ is modulated by $\mathcal{E}_{indrct}(T_o)$, and Eq.(5) says $\mathcal{E}_{indrct}(T_o)$ is the ratio of the fractional change of $\widehat{\epsilon}_{indrct}$ (or $\widehat{\epsilon}_{earth}$) caused by a given fractional change of T_o (while X is held fixed at X_o). For small T deviations from T_o, Eq.(5) can be integrated to yield:

$$\frac{\widehat{\epsilon}_{indrct}(T,X_o)}{\widehat{\epsilon}_{indrct}(T_o,X_o)} \approx \left(\frac{T}{T_o}\right)^{\mathcal{E}_{indrct}(T_o)}. \quad (6)$$

Thus, for small ΔT, Eq.(2) can be rewritten as:

$$F_{earth}^{out}(T, X_o) = \sigma \frac{\widehat{\epsilon}_{indrct}(T_o, X_o)}{T_o^{\mathcal{E}_{indrct}(T_o)}} T^{4+\mathcal{E}_{indrct}(T_o)}, \quad \Delta T << T_o. \tag{7}$$

In other words, $4 + \mathcal{E}_{indrct}(T_o)$ is the *effective* temperature exponent of the Stefan-Boltzmann formula for the Earth's outward radiation.

The value of $\mathcal{E}_{indrct}(T_o)$ depends on indirect effects of changes in T on $\widehat{\epsilon}_{indrct}(T, X_o)$ via the natural physics of the Earth's atmosphere—e.g. water vapor feedback, etc. But *the restricted wave-length window of the outward infrared radiation also plays a role*, as will be shown in §3 of this paper.

2 The IPCC $\mathcal{E}_{indrct}(T_o)$

The *Intergovernmental Panel on Climate Change* (IPCC) recently issued several well-publicized reports [1, 2, 3]. The value of $\mathcal{E}_{indrct}(T_o)$ can be extracted from these reports.

2.1 The IPCC $\Delta F_{CO_2}^{out}$

In its 2001 and 2007 reports, IPCC recommended the following "simplified expression" for $\Delta F_{CO_2}^{out}$ [2, see its Table 6.2] which is attributed to direct carbon dioxide radiation absorption:

$$\Delta F_{CO_2}^{out} \approx 5.35 \ln \frac{X}{X_o} \quad \mathit{watts/meter}^2. \tag{8}$$

The pre-industrial value X_o was 285 parts per million, or 285 ppm. Eq.(8) was first reported by Myhre *et. al.* in 1998 [4].

2.2 The IPCC Climate Sensitivity

The IPCC recommendation for ΔT is [1]:

$$\Delta T = \frac{\beta}{\ln 2} \ln \frac{X}{X_o} \quad {}^oC, \tag{9}$$

where β is called the Earth's equilibrium *climate sensitivity*—the total (direct and indirect) surface temperature rise ΔT (above the pre-industrial value T_o) when the atmospheric carbon dioxide concentration X reaches twice its pre-industrial value X_o.

The IPCC's "likely range" for β is between 2 ^oC and 4.5 ^oC; its "very unlikely" range for β is below 1.5 ^oC; its "best estimate" is $\beta \approx 3$ ^oC.

2.3 Extracting the IPCC $\mathcal{E}_{indrct}(293\ ^oK)$

Using Eq.(8), IPCC's $\Delta F^{out}_{CO_2}$, in Eq.(4), one obtains :

$$\frac{\Delta T}{T_o} \approx \frac{1}{4 + \mathcal{E}_{indrct}(T_o)} \frac{5.35}{F^{out}_{earth,o}} \times \ln \frac{X}{X_o}. \tag{10}$$

A formula for $\beta(T_o)$ can be obtained by comparing Eq.(9) and Eq.(10):

$$\beta(T_o) = \frac{\ln 2}{4 + \mathcal{E}_{indrct}(T_o)} \, \frac{5.35}{\sigma \widehat{\epsilon}_{earth}(T_o, X_o) T_o^3} \quad {}^oC. \tag{11}$$

Using $T_o \approx 20\ ^oC = 293\ ^oK$ and $\widehat{\epsilon}_{earth}(293\ ^oK, X_o) = 0.82$, one has:

$$\beta(293\ ^oK) = \overbrace{\mu(\mathcal{E}_{indrct})}^{\text{multiplier}} \times \overbrace{0.79}^{\text{direct}} \quad {}^oC, \tag{12}$$

where the dimensionless $\mu(\mathcal{E}_{indrct})$ is a *multiplier of direct effect*:

$$\mu(\mathcal{E}_{indrct}) \equiv \frac{4}{4 + \mathcal{E}_{indrct}(T_o)}. \tag{13}$$

The IPCC's likely range of $\beta(T_o)$ implies $5.7 > \mu(\mathcal{E}_{indrct}) > 1.9$, and thus its best estimate of $\beta(T_o) \approx 3\ ^oC$ implies best estimates for $\mu(\mathcal{E}_{indrct}) \approx 3/0.79 = 3.8$, and $\mathcal{E}_{indrct}(T_o) \approx -2.95$. This is consistent with Hall and Manabe [5, see its §7.2.1.1] who suggested that "water vapour feedback alone can lead to 3.5 times as much warming as would be the case if water vapour concentration were held fixed."

Thus, the IPCC's best estimate for the effective temperature exponent of the Stefan-Boltzmann formula for the Earth's F^{out}_{earth} is $4 - 2.95 = 1.05 \approx 1$ (see Eq.(7)).

3 An Explicit Formula for $\hat{\epsilon}_{earth}(T, X)$

The value of $F^{out}_{earth}(T, X)$ can be directly evaluated by:

$$F^{out}_{earth}(T, X) = \int_0^\infty \epsilon(\lambda, T; X)\mathcal{P}(\lambda, T)d\lambda, \tag{14}$$

where $\epsilon(\lambda, T; X)$ is the *wave-length resolved effective emissivity* of the Earth when viewed from the top of the Earth's atmosphere, λ is wave length (meters), $\mathcal{P}(\lambda, T)$ is the classical Planck's black body radiation intensity:

$$\mathcal{P}(\lambda, T) = \frac{2\pi hc^2}{\lambda^5(\exp(\frac{hc}{\lambda kT}) - 1)} \quad watts/meter^2 \text{ per wave length,} \tag{15}$$

h is Planck's constant, c is the speed of light, and k is the Boltzmann constant. Fig. 1 shows $\mathcal{P}(T)$ versus λ for $T = 288\ ^oK$ ($15\ ^oC$) and $293\ ^oK$ ($20\ ^oC$).

For fixed T, $\mathcal{P}(\lambda, T)$ peaks at λ_{peak}. The Wien's Displacement Law is:

$$T\lambda_{peak} = 2.9 \times 10^{-3} \quad ^oK\ meter. \tag{16}$$

It is seen that for $T \approx 293\ ^oK$, the peaks are near $\lambda_{peak} \approx 10^{-5}$ meters, or approximately 10 microns.

It is convenient to rewrite $\mathcal{P}(\lambda, T)d\lambda$ as follows:

$$\mathcal{P}(\lambda, T)d\lambda = T^4\mathcal{B}(\xi)d\xi, \tag{17a}$$

$$\xi \equiv T\lambda, \tag{17b}$$

$$\mathcal{B}(\xi) \equiv \frac{2\pi hc^2}{\xi^5(\exp(\frac{hc}{k\xi}) - 1)}. \tag{17c}$$

Eq.(14) can now be written as:

$$F^{out}_{earth}(T, X) = T^4 \int_0^\infty \epsilon(\lambda = \xi/T, T; X)\mathcal{B}(\xi)d\xi. \tag{18}$$

Note that by definition $\sigma \equiv \int_o^\infty \mathcal{B}(\xi)d\xi$.

Comparing Eq.(18) with Eq.(2), one obtains an explicit formula for $\widehat{\epsilon}_{earth}(T, X)$:

$$\widehat{\epsilon}_{earth}(T, X) = \frac{1}{\sigma} \int_0^\infty \epsilon(\lambda = \xi/T, T; X)\mathcal{B}(\xi)d\xi. \tag{19}$$

Credible data on the wave-length resolved effective emissivity $\epsilon(\lambda, T; X)$ at the top of the Earth's atmosphere is needed to evaluated $\widehat{\epsilon}_{earth}(T; X)$.

3.1 Single Main Window Approximation

Fig. 2 shows the Earth's wave-length resolved $\epsilon(\lambda, T; X)$ calculated by integrating the absorption cross-sections from the HITRAN 2000 database [6] across a 1976 standard atmosphere. Thermal radiations from the top of the Earth's atmosphere, which is quite cold compared to T_o, have been ignored. See also Fig. 1 of Goody and Robinson [7]. It is seen that there are three windows: one between $2 - 6$ microns, one between $7 - 14$ microns, and one between $16 - 22$ microns. It is obvious that the middle window is the most important because the magnitude of $\epsilon(\lambda, T; X)\mathcal{P}(\lambda, T)$ there is much bigger than those in the other two windows. For this middle window, carbon dioxide is responsible for the long wave-length cutoff which is nominally at about 14 microns, and nitrous oxide, methane and water vapor are responsible for the short wave-length cutoff which is nominally at about 7 microns. In the interior of the middle window, water vapor continuum absorption is responsible for $\epsilon(\lambda, T; X)$ there being somewhat below unity.

Let the Earth's $\epsilon(\lambda, T; X)$ be approximated by a single main (top hat) window:

$$0 \leq \lambda < \lambda_1 \approx 7 \text{ microns} \qquad \epsilon(\lambda, T; X) \approx 0, \tag{20a}$$

$$\lambda > \lambda_2(X) \approx 14 \text{ microns} \qquad \epsilon(\lambda, T; X) \approx 0, \tag{20b}$$

$$\lambda_2(X) \geq \lambda \geq \lambda_1 \qquad \epsilon(\lambda, T; X) \approx \epsilon_{ave}(T) < 1, \tag{20c}$$

where $\epsilon_{ave}(T)$ is an *average* value of $\epsilon(\lambda, T; X)$ inside the single main window (its definition is given at the end of this section), and is independent of λ or X. The narrow ozone spike just below 9 microns inside this window is ignored. Outside this window, the atmosphere is assumed optically thick at ground level to all outward infrared radiation. For $T \approx 293\ ^oK$, Fig. 1 shows that the Earth's λ_{peak} is inside this single main window. Hence the response of $\mathcal{P}(\lambda, T)$ to small T perturbations is much stronger inside this window than outside this window. Note that λ_2 depends on X, and that $d\lambda_2(X)/dX$ is expected to be negative so that the window narrows as X increases.

With this single main window approximation, Eq.(18) and Eq.(19) for $F^{out}_{earth}(T; X)$ and $\widehat{\epsilon}_{earth}(T; X)$ can be written as follows:

$$F^{out}_{earth}(T; X) = \epsilon_{ave}(T) T^4 \int_{T\lambda_1}^{T\lambda_2(X)} \mathcal{B}(\xi) d\xi, \tag{21a}$$

$$\widehat{\epsilon}_{earth}(T; X) = \frac{\epsilon_{ave}(T)}{\sigma} \int_{T\lambda_1}^{T\lambda_2(X)} \mathcal{B}(\xi) d\xi. \tag{21b}$$

Eq.(21a) and Eq.(18) together can be considered the definition of $\epsilon_{ave}(T)$.

3.2 Direct Effect

When X increases above X_o, the incremental direct absorption of outward infrared radiation by carbon dioxide, $\Delta F^{out}_{CO_2}$, can be calculated from Eq.(21a) by assuming some model for $d\lambda_2(X)/dX$. For example, the IPCC simplified expression for $\Delta F^{out}_{CO_2}$, displayed by Eq.(8) here, can be recovered from Eq.(21a) by assuming $d\lambda_2(X)/d\ln X$ to be a (negative) constant. No assumption of $(X - X_o)/X_o << 1$ is needed.

3.3 Indirect Effects

The indirect effect can be studied by using $\widehat{\epsilon}_{earth}(T;X)$ from Eq.(21b) to evaluate $\mathcal{E}_{indrct}(T)$ via Eq.(5). One obtains:

$$\mathcal{E}_{indrct}(T_o) = \left(\frac{\partial \ln \widehat{\epsilon}_{earth}(T;X)}{\partial \ln T}\right)_o = \mathcal{E}_{atm} + \mathcal{E}_{wndw}, \tag{22}$$

where

$$\mathcal{E}_{atm} \equiv \left(\frac{\partial \ln \epsilon_{ave}(T;X)}{\partial \ln T}\right)_o, \tag{23a}$$

$$\mathcal{E}_{wndw} \equiv \frac{[\xi \mathcal{B}(\xi)]_{\xi_2 = T_o\lambda_2(X_o)} - [\xi \mathcal{B}(\xi)]_{\xi_1 = T_o\lambda_1}}{\int_{T_o\lambda_1}^{T_o\lambda_2(X_o)} \mathcal{B}(\xi) d\xi}. \tag{23b}$$

The first term in Eq.(22), $\mathcal{E}_{atm}$, represents the response of the atmosphere. It includes the so-called *water vapor feedback effect* [5] and is expected to be negative since a warmer atmosphere can hold more water vapor—which is the Earth's dominant greenhouse gas. *Compare* Eq.(23a) *with* Eq.(5). The second term in Eq.(22), $\mathcal{E}_{wndw}$ (which can be either positive or negative) is a contribution from the restricted wave-length window. While $\mathcal{E}_{wndw}$ can be straightforwardly computed numerically from Eq.(23b), it is of interest to note that it can also be estimated *graphically*: the denominator is the area under the $\mathcal{B}(\xi)$ curve between ξ_1 and ξ_2, and the numerator is the difference of the areas of two rectangles each with its upper right corner pinned to the $\mathcal{B}(\xi)$ curve (and their lower left corners pinned at the origin). Since $\xi_2 > \xi_1$ always, then $\mathcal{B}(\xi_2) > \mathcal{B}(\xi_1)$ is sufficient to guarantee $\mathcal{E}_{wndw} > 0$. This inequality is in fact true for the Earth with $T_o \approx 293\ ^oK$. Using graphical estimation on Fig. 1, one obtains $\mathcal{E}_{wndw} \approx +1$ for the Earth, a number that is essentially independent of all the poorly understood details in the atmosphere.

The Earth's $\epsilon(\lambda, T_o; X_o)$ data shown in Fig. 2 is the sole rationale for the single main top hat window approximation. The $\mathcal{E}_{wndw}$ term exists because the amount of infrared radiation that can pass through a fixed wave-length top hat window is T dependent. Its sign and magnitude depends only on the location of λ_{peak} relative to this window. No other physics is involved.

If multiple top hat windows were used to approximate $\epsilon(\lambda, T; X)$, the resulting $\mathcal{E}_{wndw}$ would be the weighted sum of the individual $\mathcal{E}_{wndw}$'s—the weighting factor being the fractional contribution of each top hat window to the total flux integral.

3.4 Comments on $\mathcal{E}_{atm}$

The term $\mathcal{E}_{atm}$ in Eq.(22) includes all other relevant effects not included in $\mathcal{E}_{wndw}$ (which arose solely from the restricted wave-length window). Since water vapor continuum absorption is mainly responsible for the magnitude of ϵ_{ave} in the top hat window, it is reasonable to associate $\mathcal{E}_{atm}$ with the water vapor feedback effect. The sign of $\mathcal{E}_{atm}$ is intuitively certain, but its magnitude is very difficult to pin down because it definitely depends on the detailed quantitative *average* response of many very difficult-to-model properties of the atmosphere to a small change of the average temperature.

The IPCC best estimate numbers imply $\mathcal{E}_{indrct} \approx -2.95$. Using this value in Eq.(22), one obtains the implied value of $\mathcal{E}_{atm}$:

$$\mathcal{E}_{atm} = -2.95 - \mathcal{E}_{wndw} \approx -4. \tag{24}$$

Thus $\mathcal{E}_{wndw} \approx +1$ can play a significant role in ameliorating the negative $\mathcal{E}_{atm}$ of the Earth's atmosphere.

4 Support from Observational Data?

It must be emphasized that the long term energy balance equation, Eq.(1), is valid only under *quasi-steady* conditions. In other words, the time dependence of all variables has been neglected. Thus it is not possible to extract the value of $\mathcal{E}_{indrct}(T_o)$ from "unsteady" observational data. For example, the Earth's orbit around the Sun has a small eccentricity, and the value of F^{in}_{solar} is approximately 7% higher in January than in July. Using 0.07 for fractional F^{out}_{earth} change in Eq.(4), one obtains $\Delta T \approx 21/(4 + \mathcal{E}_{indrct}(T_o))$ ^oC between January and July—which is much too large (even with $\mathcal{E}_{atm} \approx 0$). There exists no observational annual data to support this unreasonable ΔT estimate. Thus no useful inference on $\mathcal{E}_{indrct}(T_o)$ or $\mathcal{E}_{atm}$ can be drawn here.

The value of $F^{out}_{CO_2}$ was negligible before the industrial revolution, and it grew "slowly" to become a significant number mainly in the late 20th century. The value of X/X_o at the start of the 21st century is approximately 1.3, and the observed ΔT so far is approximately 0.7 ^oC. Assuming the quasi-steady approximation to be valid, then Eq.(9) yields $\beta \approx 0.7 * \ln 2/\ln 1.3 = 1.85$—a value slightly below the low end of the IPCC "likely range" [1, 2, 3]. The effective temperature exponent $4+\mathcal{E}_{indrct}(T_o)$ is then approximately 2.2, more than twice the IPCC best estimate value given at the end of §2.3.

Let R denote the distance of a planet from the Sun. Setting $F_{solar}^{in} \propto 1/R^2$, using 4 for the Stefan-Boltzmann temperature exponent and assuming all solar planets have the same $\widehat{\epsilon}_{earth}$, one can readily recover from Eq.(3b) the textbook result that the surface temperatures of solar planets are proportional $1/R^{1/2}$. This well-known R-scaling has been observationally confirmed for most planets (on log-log plots). The glaring discrepancies are Venus and the Earth—their observed surface temperatures are significantly higher than their theoretical temperatures. Both discrepancies are explained by the carbon dioxide greenhouse effect. The atmosphere of Venus is 95% carbon dioxide, has very little water vapor, and its surface pressure is about 90 times that of the Earth. At Venus' observed temperature $\approx 740\ ^oK$, the value of $\lambda_{peak}(740\ ^oK)$ is just below 5 microns. If Fig. 2 is assumed to be a valid approximation for Venus also, then the single main emissivity window (between 7 and 14 microns) is now on the longer wave length side of the $\mathcal{P}(740\ ^oK)$ peak. Thus, Venus' $\mathcal{E}_{wndw}$ for this window is expected to be negative,[1] and the direct greenhouse effect of her atmosphere is exacerbated. Of course, when $\lambda_{peak}(740\ ^oK)$ is near 5 microns, Fig. 2 shows that the window between 2 and 6 microns can no longer be ignored and must be accounted for properly.

5 Summary

The Earth does not radiate as a grey body. Its *effective Stefan-Boltzmann temperature exponent* is $4 + \mathcal{E}_{indrct}(293\ ^oK)$, and its multiplier of direct effect is $\mu(\mathcal{E}_{indrct}) = 4/(4 + \mathcal{E}_{indrct}(293\ ^oK))$. The all important $\mathcal{E}_{indrct}(293\ ^oK)$

[1] Using the Rayleigh-Jeans Law for long wave lengths, one can analytically show that $\mathcal{E}_{wndw}(T; \lambda_1 >> hc/kT, \lambda_2 > \lambda_1) \rightarrow -3$.

is the sum of two terms: $\mathcal{E}_{atm} < 0$ which represents all the atmospheric indirect effects such as water vapor feedback and $\mathcal{E}_{wndw}$ which depends on the location of $\lambda_{peak}(T_o)$ relative to the restricted wave-length window. For $T_o \approx 293\ ^oK$, the Earth's λ_{peak} is comfortably inside the single main window for outward radiation, and $\mathcal{E}_{wndw}$ is estimated to be approximately $+1$—thus it ameliorates the impact of $\mathcal{E}_{atm} < 0$.

Unlike carbon dioxide, water vapor in the Earth's atmosphere is not "well mixed" either spatially or in time. Direct estimates of $\mathcal{E}_{atm}$ would involve some very difficult atmospheric modelings (cloud physics, albedo, . . . , etc.) and also some very difficult issues in spatial and temporal statistics. The present analysis highlights the $\mathcal{E}_{wndw}$ contribution, and provides a simple graphical way to estimate its sign and magnitude. It is shown that $\mathcal{E}_{wndw}$ is positive for the Earth, and is negative for Venus which is known for its very large greenhouse warming effects.

The bottom line is: $\mathcal{E}_{indrct}(T_o)$ is $\mathcal{E}_{atm} + \mathcal{E}_{wndw}$, and *not* $\mathcal{E}_{atm}$ *alone.* Most modern water vapor feedback models use computer simulations, and as such the question on whether the specific radiation physics in $\mathcal{E}_{wndw}$ are properly accounted for is very difficult to answer. If they are not, then their calculated magnitudes of indirect effects such as water vapor feedback—and thus their calculated equilibrium climate sensitivity to carbon dioxide—would be on the high side.

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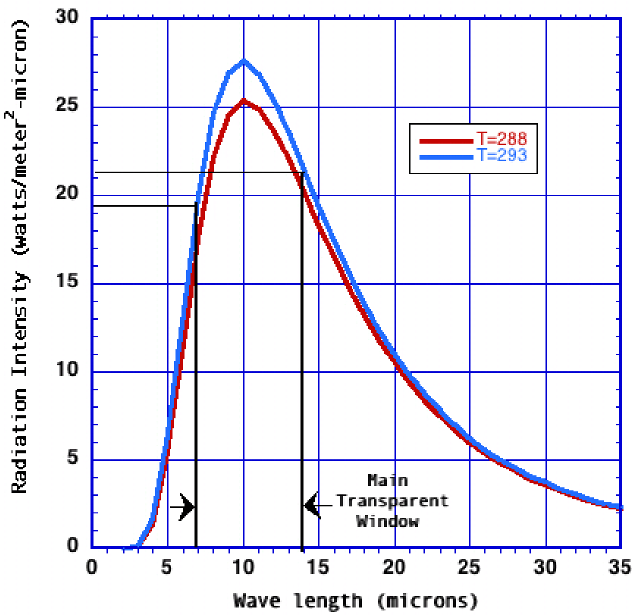

Figure 1: Planck's Blackbody Radiation vs Wave Length ($T \approx 288 - 293\,^oK$).

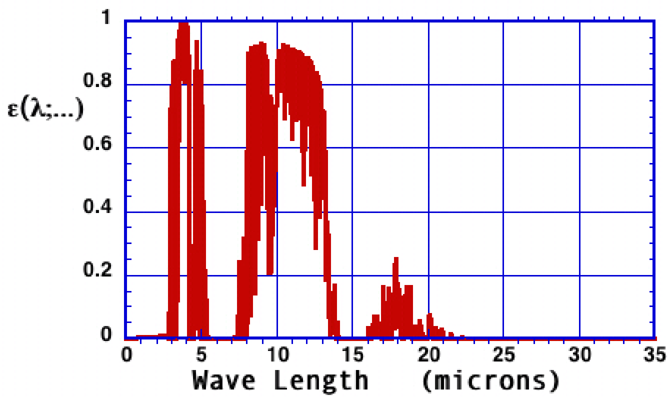

Figure 2: Earth's Effective Emissivity $\epsilon(\lambda, \ldots)$ vs Wave Length (microns)
Greenhouses gases included: $H_2O, CO_2, CH_4, O_3, NO_2, NO, N_2O_5, SF_6$.
Note that $\epsilon(\lambda; \ldots)$ inside the main window ($7 < \lambda < 14$ microns) is somewhat below unity—mainly due to water vapor continuum absorption.